An ADC primer

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1 Why do we need ADCs?

Today we can place more than a billion transistor (1 000 000 000) on a chip. The majority of transistors are used for digital processing and all digital processing needs input.

If the input is already digital — a digital photograph, a music file, an excel sheet, or a video clip — the chip can start processing right away. If the information is from the real world — the output from a crash accelerometer, a video signal, a radio signal, a signal from a microphone, the signal from an ultrasound probe, or the signal from a electrocardiogram (ECG) — it must be converted to digital.

The real world is analog — continuous in time and continuous in value. If you move your arm up and down you will notice that it does not jump from one place to another, but move in a continuous motion. This is continuous in value. Continuous in time means that the movement is not a sequence of snapshots.

Digital information is discrete in time and discrete in value. If you move your arm up and down in a digital world, the arm will jump from one point to another, and it will be impossible to place it between two points. In addition, the arm will only jump at specified times.

To convert information from analog to digital we use an analog-to-digital converter (ADC). The ADC turns continuous time to discrete time, and continuous value to discrete value.

The ADC divide the continuous values into a number of discrete levels. It's like rounding a floating point number (1.1 or 1.1204058) to an integer (1). The number of levels in an ADC are specified by the number of bits (B). A 6-bit ADC has $2^6 = 64$ levels and a 16-bit ADC has $2^{16} = 65536$ levels. The number of levels is called the resolution and determine the accuracy. ADCs range from 1-bit resolution to 24-bit resolution.

The ADC turns continuous time into discrete time by sampling. Think of a Black Moor (a goldfish) swimming back and forth in an aquarium. You have marked 32 points on the glass to mark the horizontal position. At the beat of a metronome you write down where the fish is. That is sampling, only measuring something at timed intervals.

In an ADC the intervals are determined by the sampling frequency, written in samples per second (S/s). ADCs range from 1S/s to 40GS/s (40 000 000 000 samples per second).

It would be nice to have a single 24-bit 40GS/s ADC to cover all applications, but it can't be done. Not because we don't know how to do it, but because the Heisenberg Uncertainty Principle says it's impossible [1]. Even if it could be done we would not want to because a 24-bit 40GS/s ADC will consume 2.8

million watts ¹, 500 times more than an electric stove. At that power dissipation my cellphone battery would last $3.6 \text{ms} (0.0036 \text{ s}).^2$

This paper give an introduction to ADCs and is organized as follows: Section 2 cover noise and distortion phenomena in ADC converter. Section 3 detail the measures and abbreviations used when talking about ADCs. In Section 4 we discuss one of ADC architectures featured in this thesis, the pipelined ADC.

For a deeper introduction to data converters we suggest reading [2], or [3].

2 Limiting factors for ADC accuracy

2.1 Noise

Noise is present in any analog system. Noise sources are often divided into two categories: *intrinsic* and *extrinsic*. *Intrinsic* refer to an inherent property of the system. *Extrinsic* refers to an external influence. In this section we will describe the *Intrinsic* phenomena of noise.

Noise manifests as random fluctuation of a signal. There are three main noise sources: thermal noise, shot noise and flicker noise. Thermal noise stem from the random fluctuation of charge carriers, shot noise from charge carriers moving across a potential barrier and flicker noise from the random trapping and release of charge carriers. Thermal noise and flicker noise are the dominating noise sources in MOSFET transistors. The dominating noise source in highspeed analog-to-digital converters (ADCs) is thermal noise. For a comprehensive treatise on noise phenomena we refer to Aldert Van Der Ziel's "Noise in Solid State Devices and Circuits" [4].

Noise place a lower limit on the resolution of a system. The systems discussed in this thesis are switched-capacitor circuits which have an accumulated noise

$$C = \frac{48kT2^{2B}}{V_{PP}^2}$$
(1)

where k is Boltzmann's constant, T is the temperature in Kelvin, B is the number of bits, and V_{PP} is the peak-to-peak signal swing.

If we assume a transconductance amplifier, it needs a transconductance of

$$g_m = 2\pi C f_s \tag{2}$$

And a current of

$$I = \frac{1}{2}gm \times V_{EFF} \tag{3}$$

where V_{EFF} is the effective overdrive of the transistor that provides the transconductance.

If we assume $V_{EFF} = V_{DD}/10$, $V_{PP} = \frac{1}{2}V_{DD}$, and $P = V_{DD}I$, where P is the power dissipation we have that

$$P = 19.2\pi k T 2^{2B} f_s \tag{4}$$

which for B = 24, T = 300 and $f_s = 40 GHz$ is P = 2.8 MW

 $^2\rm My$ cell phone battery has 780mAh. With a current drain of I=P/V=2.8MW/3.6V=778kA it would run for 780mAh/778kA=3.6ms.

¹ The required capacitance is given by

approximated by

$$V_n^2 = a_1 k T / C \tag{5}$$

where k is Boltzmann's constant (1.38×10^{-23}) , T the temperature in Kelvin, C the sampling capacitance and a_1 a constant.

2.2 Quantization errors

One of the fundamental limitations of Nyquist converters is the quantization error. Quantization of a continuous value signal is a non-linear operation. We define the output y_Q as

$$y_Q = Q(y_a) = y_a + q_e \tag{6}$$

where y_a is the input signal, Q(x) is the quantization function and q_e is the error signal due to quantization.

2.2.1 The exact solution

A

The quantization operation distort the input signal. We can write the quantization error, q_e , as

$$q_e = y_a - y_Q \tag{7}$$

If the input signal, y_a , is a ramp function, the quantization error will be a sawtooth function.

For a sinusoidal input the quantization error become more complex. For the N-bit case the quantization error can be written as [5]

$$y_Q = \sum_{p=1}^{\infty} A_p \sin p\omega t \tag{8}$$

where ω is the angular frequency, t is time and p is the harmonic index. The amplitude of the individual harmonics, A_p , is defined as

$$A_{p} = \delta_{p1}A + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_{p}(2m\pi A), p = odd$$
(9)

$$A_p = 0, p = even \tag{10}$$

where

$$\delta_{p1} = 1, p = 1 \tag{11}$$

$$\delta_{p1} = 0, p \neq 1 \tag{12}$$

and $J_p(x)$ is a Bessel function of the first kind. If we approximate the amplitude of the input signal as

$$A = \frac{2^n - 1}{2} \approx 2^{n-1} \tag{13}$$

where n is the number of bits, we can rewrite (9) as

$$A_p = \delta_{p1} 2^{n-1} + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_p(2m\pi 2^{n-1}), p = odd$$
(14)

Quantization error only contain odd harmonics of the input signal. The expression in (14) is complex to calculate, and is not suited for a quick approximation of the quantization error.

2.2.2 The approximation

It is generally accepted that for sufficient quantization steps (enough bits) and an active input signal the quantization error, q_e , can be approximated by a white noise [6]. The quantization error varies between $-\frac{1}{2}LSB < q_e < \frac{1}{2}LSB$ and has an average power of $\overline{q_e^2} = \frac{1}{12}LSB^2$. The quantization error place a fundamental limit of the resolution of a Nyquist converter with a finite number of bits. The general expression of signal to noise ratio is

$$SNR = 6.02B + 1.76dB$$
 (15)

where B is the number of bits and we assume a sinusoidal input signal.

2.2.3 The exact solution versus the approximation

A more accurate expression for the dynamic range than (15) of a n-bit converter, derived from (8) and (14), is

$$SNR = \frac{2^{n-1} + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_1(2m\pi 2^{n-1})}{\sqrt{\sum_{i=1}^{\infty} [\sum_{m=1}^{\infty} \frac{2}{m\pi} J_{2i+1}(2m\pi 2^{n-1})]^2}}$$
(16)

In Table 2.2.3 the SNR for 1 to 10 bits in the quantizer is shown [3]. The approximation (15) overestimates the signal-to-noise ratio. The overestimation is reduced with a higher number of bits.

2.3 Sampling clock jitter

Sampling is controlled by a clock signal. The clock signal has a frequency (f_s) called the sampling frequency. According to the sampling theorem [7], signal frequencies at, or below, $\frac{1}{2}f_s$ can be accurately reproduced from the sampled data.

How accurate a signal can be sampled depend on the sampling time uncertainty called jitter or clock phase noise. Jitter is a random fluctuation of the sampling instance. The source of jitter is usually thermal noise in clock buffer-, amplifier- or generator-circuits [3].

The effect of jitter is more noise. The results can be seen in Fig. 1 and Fig. 2. In Fig. 1 the sampled spectrum with and without jitter is shown. Note

| Number of bits | Accurate SNR | Approximate SNR | Percent error |
|----------------|--------------|-----------------|---------------|
| 1 | 6.31 | 7.78 | 18% |
| 2 | 13.30 | 13.80 | 6% |
| 3 | 19.52 | 19.82 | 3.5% |
| 4 | 25.59 | 25.84 | 2.9% |
| 5 | 31.65 | 31.86 | 2.4% |
| 6 | 37.70 | 37.88 | 2% |
| 7 | 43.76 | 43.90 | 1.6% |
| 8 | 49.82 | 49.92 | 1.1% |
| 9 | 55.87 | 55.94 | 0.8% |
| 10 | 61.93 | 61.96 | 0.3% |
| | | | |

Table 1: SNR as function of the number of bits.

that the signal without jitter has finite resolution because we have added noise to emulate quantization noise. The jitter is simulated as a random fluctuation of the sampling instant. From Fig. 1 we can see that noise power is increased when jitter is added. In Fig. 2, signals are shown in time domain. We can see that the signal with jitter samples an incorrect value from the input signal.



Figure 1: Spectrum with and without jitter in sampling of a sinusoid

It is possible to derive equations for the maximum jitter that can be tolerated in an ADC. As we can see from Fig. 2, at a specific time t we sample a value A without jitter, and with jitter we sample a value $A + \Delta A$. For the jitter not to have an adverse effect on the resolution of the converter, the factor ΔA must be less than the quantization step of the converter. An expression for the



Figure 2: Time domain plot with and without jitter in sampling of a sinusoid

maximum jitter, Δt_{max} , can be written as (17) [3], where B is the number of bits and f_{in} is the maximum input frequency.

$$\Delta t_{max} = \frac{1}{\pi f_{in} 2^B} \tag{17}$$

Since the amount of jitter depend on the input signal frequency, as shown in (17), it is imperative that clock amplifiers/buffers in high-speed ADCs are designed to have sufficiently low jitter.

For a 10-bit pipelined ADC with a 50MHz input the maximum jitter is 6.2ps, which is trivial to achieve. For a 15-bit ADC with a 15MHz input frequency the maximum jitter is 0.65ps, which is hard to achieve. The lowest published jitter (that we could find) in a ADC is 50 femto-seconds $(50 \times 10^{-15} s)$ [8].

2.4 Distortion

The output, y_{out} , of a ADC for a sinusoidal input can be written as

$$y_{out} = f(x), x = A\cos(\omega t) \tag{18}$$

where f(x) is the system function, A is the amplitude, t is time and ω is the angular input frequency. For a linear ADC f(x) is approximated by

$$f(x) = x + e_n \tag{19}$$

where e_n is a noise component. Thus the output will be

$$y_{out} = A\cos(\omega t) + e_n \tag{20}$$

A real multi-bit ADC is non-linear. If the system function is weakly non-linear we can approximate f(x) using a Taylor series expansion. In this example we will use a Taylor series expansion around zero. The system function f(x) then becomes

$$f(x) = K_1 x + K_2 x^2 + K_3 x^3 + \dots + K_i x^i + e_n$$
(21)

where the coefficients K_i is given by

$$K_i = \frac{1}{i!} \frac{d^i f(0)}{dx} \tag{22}$$

We can calculate the output as a function of the input using (21); we will only include the first three terms.

$$y_{out} = K_1 A \cos(\omega t) + K_2 A^2 \cos^2(\omega t) + K_3 A^3 \cos^3(\omega t) + e_n$$
(23)

By using the well know relation

$$\cos a \cos b = \frac{1}{2} [\cos(a-b) + \cos(a+b)]$$
 (24)

we can rewrite (23) as

$$y_{out} = K_1 A \cos(\omega t) + \frac{K_2 A^2}{2} [1 + \cos(2\omega t)] + \frac{K_3 A^3}{4} [3\cos(\omega t) + \cos(3\omega t)]$$
(25)

For a weakly non-linear system with a single sinusoid input signal we will have harmonics in the output signal at $n\omega$ where n is an integer. If we have two or more sinusoidal input signals there will, in addition to harmonics, be intermodulation products at $k\omega_1 \pm n\omega_2$, where ω_1 and ω_2 are the input signal frequencies and k and n are integers.

Since most analog and mixed signal integrated circuits use differential signaling it is useful to know how distortion behaves in a differential circuit. In addition to improve signal to noise ratio ³, a differential system suppress even order distortion. The output of a differential circuit can be defined as:

$$y_{out} = f_1(x) - f_2(-x) \tag{26}$$

where $f_k(x)$ are the individual non-linear transfer functions for the differential paths. We define $f_k(x)$ as

$$f_k(x) = K_{0k} + K_{1k}x + K_{2k}x^2 + K_{3k}x^3$$
(27)

$$SNR = \frac{2A}{\sqrt{2}e_n} = \sqrt{2}\frac{A}{e_n}$$

³Signals add linearly when combined after a differential system. A sinusoid with an amplitude of A in the differential paths will have an amplitude of 2A after combination, as shown by (29). Assuming uncorrelated noise sources in the two differential paths the output noise power would be $e_{nout}^2 = e_{n1}^2 + e_{n2}^2$, where e_{n1}^2 and e_{n2}^2 are the noise powers of the differential paths. If the noise sources have the same power the output root mean square will be $e_{nout} = \sqrt{2}e_n$. Thus, the signal to noise ratio improves with a factor of $\sqrt{2}$, since

where K_{ik} are the distortion coefficients defined in (22). K_{0k} is the zero order distortion (DC) resulting from for example offsets. When we calculate the output y_{out} we get

$$y_{out} = K_{01} - K_{02} + [K_{11} + K_{12}]x + [K_{21} - K_{22}]x^2 + [K_{31} + K_{32}]x^3$$
(28)

If $K_{i1} = K_{i2}$ the equation reduce to

$$y_{out} = 2K_{11}x + 2K_{31}x^3 \tag{29}$$

Equation (29) proves that even order distortion is removed if the distortion in the in differential paths are equal.

3 Abbreviations and measures

For a thorough definition of the different abbreviations and measures we refer to chapter 1 and 2 in [3]. This chapter summarizes some of the measures used when ADCs are discussed.

3.1 MSB and LSB

MSB is the Most Significant Bit and LSB is the Least Significant Bit. The LSB of a ADC is equal to the converter step.

3.2 INL and DNL

Fig. 3 shows an example of INL and DNL. INL is the Integral Non-Linearity of a ADC. It is the deviation of the quantization steps from a straight line when linear errors (offset and gain errors) are removed.

DNL is the Differential Non-Linearity of a ADC. It describes the difference between two neighboring analog threshold of the ADC.

3.3 SNR

SNR is the Signal-to-Noise Ratio of a system. It is defined as

$$SNR = 10\log\frac{Signal\ power}{Noise\ 'power} \tag{30}$$

3.4 SFDR

SFDR is Spurious Free Dynamic Range. In a FFT it is the difference between the power of the signal and the most powerful harmonic.



Figure 3: INL and DNL. Units in LSB

3.5 ENOB

ENOB is Effective Number Of Bits. If we have the measured SNR of an ADC we can use (15) to get effective number of bits:

$$ENOB = \frac{SNR - 1.76}{6.02}$$
 (31)

It should be noted that in data sheets where SNR is given it is sometimes measured without the power of the first six harmonics. We would get a more accurate ENOB if we included distortion in the SNR. SNR with distortion is often named SNDR (Signal to Noise and Distortion Ratio) or SINAD (Signal to Noise And Distortion).

3.6 ERBW

ERBW is Effective Resolution BandWidth. It is defined as the bandwidth where the SNR (preferably with distortion) of the ADC stays within 3dB.

3.7 Aliasing

Aliasing is the folding of signal frequencies higher than the Nyquist frequency $f_s/2$ into the base-band. Aliasing is mostly an unwanted phenomena. To avoid aliasing an anti-alias filter is used. This can be a pure analog filter in front of the ADC, or a combination of analog filtering and digital post filtering. Note that digital post filtering, in other words filtering after sampling, requires a certain oversampling of the base-band. If the base-band is limited to f_b the sampling frequency might be at $8f_b$ [3].

4 Pipelined ADC

The pipelined ADC is used for high-speed applications (1MS/s - 1GS/s) with medium to high resolution (8-bit - 15-bit).

A block diagram of a pipelined ADC can be seen in Fig. 4. The pipelined ADC is built from multiple stages, normally preceded by a sample-and-hold (S/H) circuit. In each stage B-bits are determined. A B-bit ADC, called a sub-ADC (SADC), quantize the input signal to the stage. The quantized signal is subtracted from the input using a B-bit DAC. The residue after subtraction is amplified by $Gain = 2^B$ so the input swing of the pipelined stage is equal to the output swing.

Pipelined ADCs use an over-range to correct for some of the non-idealities in SADC and amplifier. Hence, the sum of the B-bits from each stage is larger than the resolution of the overall ADC. The most common pipelined stage has 1.5-bits (two comparators).

In the first stage the DAC and amplifier (Gain) need to have full accuracy. In the subsequent stages the required accuracy is lower due to the accumulated gain. Therefore, stages 2 through stage n are usually scaled in order to reduce the power dissipation of these stages.

The number of bits selected for each stage, B, depends on the overall resolution of the ADC and what is possible to implement within the restrictions of the processing technology.

4.1 Speed of pipelined ADCs

By pipelining stages, the speed of the converter can be equal to the maximum speed of each stage. Stages in a pipelined ADC normally have two phases: sampling and amplification.

Stages are clocked with opposite phases, so stage 1 amplifies while stage 2 samples. If the clock period of the overall converter is T_s , then each stage has $T_s/2$ for each phase. Sampling normally occurs at the end of a phase, thus stage 1 has $T_s/2$ to settle before stage 2 samples its output signal.

A new sample is available at the output of the pipelined ADC at the end of each clock period. Although the pipelined ADC has a large throughput, the latency (Latency is the time it takes from the analog input signal is sampled to the digital word is available at the output of the ADC) depend on the number of



Figure 4: Block diagram of a pipelined ADC

stages. This excludes the pipelined ADC from some applications where latency is key, for example as a quantizer in a conventional $\Sigma\Delta$ ADC.

4.2 The pipelined stage

The most common pipelined stage has 1.5-bits. With 1.5-bits the SADC is implemented as a flash ADC with two dynamic comparators. An implementation of a 1.5-bit stage is shown in Fig. 5.

In the SADC there are two comparators with their thresholds at $\pm V_r/4$, where V_r is the reference voltage. At the end of the sample phase the comparators quantize the input signal. At the same time the input signal is sampled onto capacitors C_1 and C_2 . An advanced clock phase (p_{1a}) is used to reduce signal dependent charge injection from p_1 switches.

In the multiplication phase the quantized input signal is used to decide which of the voltages $-V_r = V_{RN}$, $V_{CM} = 0$ or $V_r = V_{RP}$ should be connected to the capacitor C_1 . The capacitor C_2 is connected to the opamp output during the amplification phase. The opamp forces virtual ground at its negative input, hence the voltage across capacitor C_1 is zero if we assume C_1 is connected to $V_{CM} = 0$. The charge of C_1 is transferred to C_2 . When the opamp has settled the output signal is ready to be sampled by the next stage. The output of a stage can be written as

$$(1 + \frac{C_1}{C_2})V_{input} - V_r, \qquad V_{input} > \frac{V_r}{4}$$

$$(32)$$

$$v_{residue} = \begin{cases} (1 + \frac{C_1}{C_2})V_{input}, & -\frac{V_r}{4} < V_{input} < \frac{V_r}{4} \end{cases}$$
(33)

$$\left((1 + \frac{C_1}{C_2})V_{input} + V_r, \quad V_{input} < -\frac{V_r}{4} \right)$$
(34)

The digital output can be encoded in many ways, we use

$$\int 10, \qquad V_{input} > \frac{V_r}{4} \tag{35}$$

$$p_{out} = \begin{cases} 01, & -\frac{V_r}{4} < V_{input} < \frac{V_r}{4} \\ 00, & V_r < -\frac{V_r}{4} \end{cases}$$
(36)

$$\bigcup_{input} 00, \qquad V_{input} < -\frac{v_r}{4} \tag{37}$$



Figure 5: Implementation of a 1.5-bit stage

4.3 Error correction in pipelined ADCs

Assume that one of the comparators in the SADC has an offset that makes its threshold larger than $V_r/4$, for example $1.5V_r/4$. If the input value of stage 1 is $1.2V_r/4$ the SADC will incorrectly decide that the quantized word should be "01" and not "10", which would be correct. The output value of stage 1 would be $2 \times 1.2V_r/4 = 1.2V_r/2$. Stage 2 will then decide that the quantized word is "10", even if it has the same offset in the SADC. To combine stage output words they are shifted and summed leading to

| Stage 1: | 01 |
|-------------------|------|
| Stage 2 MSB: | 1 |
| Corrected Stage 1 | : 10 |

Thus a 1.5-bit pipelined stage can tolerate an offset in comparators up to $\pm V_r/4$, greatly reducing the required accuracy of the SADC.

4.4 Reducing power in pipelined ADCs

The sample-and-hold in front of the converter and the opamps in each stage dissipates most of the power in a pipelined converter. The opamp in a stage is only used during half the clock period. Techniques have been developed that switch off the opamp during half the clock period to reduce power dissipation, however this technique is normally not suitable for high-speed designs due to the slow turn on time of high-speed opamp. Another technique is to share an opamp between two stages [3].

4.5 Effects of finite gain

Finite gain in the amplifier cause leakage of the residue to the digital output. Assume a 1 bit pipelined stage with a B-bit ideal back-end. The output signal of the ADC is given by

$$y_o = D_0 + \frac{1}{2}D_B$$
 (38)

where D_0 is the digital output from the first stage and D_B is the digital output from the back-end. The digital output from the first stage can be written as

$$D_0 = v_i + e_0 \tag{39}$$

where v_i is the input signal, and e_0 is the quantization error of the first SADC. The output of the back-end can be written as

$$D_B = v_r + e_n \tag{40}$$

where v_r is the residue from the first stage, and e_n is the quantization error of the back-end.

The residue for a 1 bit stage can be written as

$$v_r = 2(1+\epsilon_q)(v_i - D_0) \tag{41}$$

where ϵ_q is the gain error.

Combining (38) through (41) yields

$$y_o = v_i + \epsilon_g e_0 + \frac{1}{2}e_n \tag{42}$$

The quantization error of the SADC leaks through to the output. This error is not white, because the quantization error of a low bit converter is not white. Accordingly, harmonics are introduced in the output of the converter.

4.6 Pipelined ADC summary

A switched capacitor amplifier, like the one used in 1.5-bit stage, requires a highgain opamp. In the first stage the gain must be higher than the resolution of the pipelined ADC. For example, for a 10-bit converter we need 68-dB gain in the first stage opamp. High-speed, high-gain opamps have high power dissipation and are difficult to implement in modern nano-scale processes. Too low gain in the opamp leads to incorrect settling of the switched-capacitor amplifier, so the gain is less than 2 for a 1.5-bit stage.

Due to the architecture of the pipelined converter this leads to non-linear distortion.

Techniques like correlated level shifting [9], open-loop residue amplifiers [10], gain calibration [11], [12] and comparator-based switched capacitor circuits (CBSC) [13] have been developed to either make the opamp easier to design, or replace the opamp completely.

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